应 用 数 学
MATHEMATICA APPLICATA
2007,20(3):505~511

# Exact Solutions to the Higher Order Nonlinear Schrödinger Equation

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**Abstract**: By using the modified mapping method and the extended mapping method, we derive some new exact solutions of the higher order nonlinear schrödinger equation, which are the linear combination of two different Jacobi elliptic functions. The solutions in the limit cases have also been studied.

**Key words**: Exact solutions; Higher order nonlinear schrödinger equation; Modified mapping method; Extended mapping method
**CLC Number**: O175.25 **AMS(2000)Subject Classification**: 35Q51
**Document code**: A **Article ID**: 1001-9847(2007)03-0505-07

## 1. Introduction

In the study of nonlinear waves, one of the fundamental objects is the travelling wave solutions which are solutions of constant from moving with a fixed velocity. Of particular interest are three types of travelling wave: the solitary waves, which are localized travelling waves asymptotically zero at large distances; the periodic waves and the kink waves, which rise or descend from one asymptotic state to another. Investigation of such solutions has been a hot topic of research for several decades. There are many methods for finding special solutions to nonlinear evolution equations. Some of the most important methods are Bäcklund transformations[1], the algebraic method[2], tanh method[3,4], Jacobi elliptic function method and its extensions[5,6], the Weierstrass elliptic function method[7] and so on.

Recently, the exact periodic wave solutions in terms of the Jacobi elliptic functions for non-linear evolution equations attract considerable interest. The three basic Jacobi elliptic functions $\mathrm{sn}\xi = \mathrm{sn}(\xi\backslash m)$, $\mathrm{cn}\xi = \mathrm{cn}(\xi\backslash m)$ and $\mathrm{dn}\xi = \mathrm{dn}(\xi\backslash m)$, where $m$ $(0 < m < 1)$ is the modulus of the elliptic functions, satisfy the well-known type of trigonometric relations such as:

$$\mathrm{sn}^2\xi + \mathrm{cn}^2\xi = 1, \mathrm{dn}^2\xi + m^2\mathrm{sn}^2\xi = 1, (\mathrm{sn}\xi)' = \mathrm{cn}\xi\mathrm{dn}\xi, (\mathrm{cn}\xi)' = -\mathrm{sn}\xi\mathrm{dn}\xi, (\mathrm{dn}\xi)' = -m^2\mathrm{sn}\xi\mathrm{cn}\xi.$$

When $m \to 1$, the Jacobi elliptic functions degenerate to the hyperbolic functions, i. e.

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* **Received date**: December 1, 2006
**Biography**: PENG Yan-ze, male, Han, Hubei, professor, major in nonlinear evolution equation.

$$\mathrm{sn}\xi \to \tanh\xi, \mathrm{cn}\xi \to \mathrm{sech}\xi, \mathrm{dn}\xi \to \mathrm{sech}\xi.$$

When $m \to 0$, the Jacobi elliptic functions degenerate to the trigonometric functions, i. e.

$$\mathrm{sn}\xi \to \sin\xi, \mathrm{cn}\xi \to \cos\xi, \mathrm{dn}\xi \to 1.$$

There are nine other Jacobi elliptic functions which can be expressed in terms of the three basic ones. The detailed explanations about it can be found in reference [8].

A mapping method and its extensions have been successfully used to obtain various Jacobi elliptic functions solutions and other important exact solutions of a new Hamiltonian amplitude equation [9~11]. In reference [12], Gong L X derived the Jacobi elliptic function solutions of the nonlinear Schrödinger equation by the modified mapping method. In this paper, using the modified and extended mapping method, we derive several new periodic wave solutions to the higher order nonlinear Schrödinger equation, which are the linear combination of two Jacobi elliptic waves, and solitary wave solutions and trigonometric periodic wave solutions. It is the first time to use these two methods to get exact solutions of the higher order nonlinear Schrödinger equation.

## 2. Exact Solutions to the Higher Order Nonlinear Schrödinger Equation

The higher order nonlinear Schrödinger equation is taken as

$$\mathrm{i}u_t + u_{xx} + \alpha \mid u \mid^2 u + \mathrm{i}[\gamma_1 u_{xxx} + \gamma_2 \mid u \mid^2 u_x + \gamma_3 (\mid u \mid^2)_x u] = 0. \tag{1}$$

This equation has important applications in physics. Under rather general conditions, we can obtain exact periodic wave solutions of it. We seek travelling wave solution to Eq. (1) in the form

$$u(x,t) = \phi(\xi)\mathrm{e}^{\mathrm{i}(Kx-\Omega t)}, \xi = k(x-ct). \tag{2}$$

Substituting Eq. (2) into Eq. (1), we have

$$\begin{aligned} &\mathrm{i}(\gamma_1 k^3 \phi''' - 3\gamma_1 K^2 k\phi' + \gamma_2 k\phi^2\phi' + 2\gamma_3 k\phi^2\phi' - ck\phi' + 2Kk\phi') \\ &+ (\Omega\phi + k^2\phi'' - K^2\phi + \alpha\phi^3 - 3\gamma_1 Kk^2\phi'' + \gamma_1 K^3\phi - \gamma_2 K\phi^3) = 0, \end{aligned} \tag{3}$$

where $k$ is assumed to be positive, and the prime meaning differentiation with respective to $\xi$. Then we have two equations

$$\gamma_1 k^2 \phi''' + (-3\gamma_1 K^2 - c + 2K)\phi' + \gamma_2\phi^2\phi' + 2\gamma_3\phi^2\phi' = 0, \tag{4}$$

$$k^2(1-3\gamma_1 K)\phi'' + (\Omega - K^2 + \gamma_1 K^3)\phi + (\alpha - \gamma_2 K)\phi^3 = 0. \tag{5}$$

Integrating Eq. (4) once and taking zero be the integration constant, we have

$$\gamma_1 k^2 \phi'' + (-3\gamma_1 K^2 - c + 2K)\phi + (\frac{1}{3}\gamma_2 + \frac{2}{3}\gamma_3)\phi^3 = 0. \tag{6}$$

Comparing Eq. (5) and Eq. (6), they have the same solutions. So the coefficients of these two equations satisfy the following equation:

$$\frac{\gamma_1 k^2}{(1-3\gamma_1 K)k^2} = \frac{2K - c - 3\gamma_1 K^2}{\Omega - K^2 + \gamma_1 K^3} = \frac{\frac{1}{3}\gamma_2 + \frac{2}{3}\gamma_3}{\alpha - \gamma_2 K}. \tag{7}$$

From Eq. (7), we can obtain

$$K = \frac{\gamma_2 + 2\gamma_3 - 3\gamma_1\alpha}{6\gamma_1\gamma_3}, \Omega = \frac{(1-3\gamma_1 K)(2K - c - 3\gamma_1 K^2)}{\gamma_1} + K^2 - \gamma_1 K^3. \tag{8}$$

If $K$ and $\Omega$ satisfy Eq. (8), then we only have to solve Eq. (6). Now in order to reduce Eq. (6). we assume

$$A = \gamma_1 k^2, B = 2K - c - 3\gamma_1 K^2, C = \frac{1}{3}\gamma_2 + \frac{2}{3}\gamma_3, \tag{9}$$

then Eq. (6) is transformed into the following one:

$$A\phi'' + B\phi + C\phi^3 = 0. \tag{10}$$

According to the modified mapping method, we assume that Eq. (10) has the solution of the form

$$\phi(\xi) = A_0 + A_1 f + B_1 f^{-1}, \tag{11}$$

where $A_i$ and $B_i$ are constants to be determined, and $f$ satisfies the following equations

$$f'^2 = pf^2 + \frac{1}{2}qf^4 + r, \tag{12}$$

where the prime denotes derivative with respect to $\xi$, and $p$, $q$ and $r$ are constants to be determined. We have established a mapping relations between the Eq. (10) and (12) through Eq. (11). We substitute Eq. (11) into Eq. (10) and make use of Eq. (12). The substitution of Eq. (11) into Eq. (10) and the use of Eq. (12) yields

$$\begin{cases} 2rAB_1 + CB_1^3 = 0, 3CB_1^2A_0 = 0, \\ pAB_1 + BB_1 + 3CB_1A_0^2 + 3CB_1^2A_1 = 0, \\ BA_0 + CA_0^3 + 6CB_1A_0A_1 = 0, \\ pAA_1 + BA_1 + 3CA_0^2A_1 + 3CB_1A_1^2 = 0, \\ 3CA_0A_1 = 0, qAA_1 + CA_1^3 = 0, \end{cases} \tag{13}$$

from which it is found that

$$A_0 = 0, A_1 = \pm\sqrt{-\frac{qA}{C}}, B_1 = \pm\sqrt{-\frac{2rA}{C}}, pA + B + 3CA_1B_1 = 0. \tag{14}$$

Using Eq. (2) and Eq. (9), we obtain the exact solutions of Eq. (6)

$$u = \left[\pm\sqrt{-\frac{3q\gamma_1}{\gamma_2 + 2\gamma_3}}kf(\xi) \pm \sqrt{-\frac{6r\gamma_1}{\gamma_2 + 2\gamma_3}}kf^{-1}(\xi)\right]e^{i(Kx+\Omega t)}, \tag{15}$$

where $f$ satisfy Eq. (12) and

$$\xi = \sqrt{\frac{\Omega - K^2 + \gamma_1 K^3}{(3\gamma_1 K - 1)(p \pm 3\sqrt{2qr})}}(x - ct), \tag{16}$$

where $K$ and $\Omega$ are given by Eq. (8), and other parameters are arbitrary constants in the sense that the expressions in the square root should be positive. And note that the choice for the positive and negative signs in Eq. (15) is arbitrary. In the following, we discuss the specific expressions of $f$ according to Eq. (12) as examples.

Case 1 $p = -2, q = 2, r = 1$. Eq. (12) has solution $f(\xi) = \tanh\xi$. Therefore we have the new solitary wave solution of Eq. (6), it also means the solution of Eq. (1).

$$u = \{\pm\tanh[k(x - ct)] \pm \coth[k(x - ct)]\} \times \sqrt{-\frac{6\gamma_1}{\gamma_2 + 2\gamma_3}}ke^{i(Kx-\Omega t)}, \tag{17}$$

where

$$k = \frac{1}{2}\sqrt{\frac{\Omega - K^2 + \gamma_1 K^3}{3\gamma_1 K - 1}} \text{ or } k = \frac{1}{2}\sqrt{-\frac{\Omega - K^2 + \gamma_1 K^3}{2(3\gamma_1 K - 1)}}.$$

Case 2 $p = -(1 + m^2), q = 2m^2, r = 1$. Now the solution of Eq. (12) is $f(\xi) = \mathrm{sn}\xi$ or $f(\xi) = \mathrm{cd}\,\xi \equiv \frac{\mathrm{cn}\,\xi}{\mathrm{dn}\,\xi}$, and we obtain the periodic wave of Eq. (1)

$$u = \{\pm m\,\mathrm{sn}[k(x - ct)] \pm \mathrm{ns}[k(x - ct)]\}\sqrt{-\frac{6\gamma_1}{\gamma_2 + 2\gamma_3}}ke^{i(Kx-\Omega t)}, \tag{18}$$

or

$$u = \{\pm m\mathrm{cd}[k(x-ct)] \pm \mathrm{dc}[k(x-ct)]\}\sqrt{-\frac{6\gamma_1}{\gamma_2+2\gamma_3}}k\mathrm{e}^{\mathrm{i}(Kx-\Omega t)}, \tag{19}$$

where

$$k=\sqrt{\frac{\Omega-K^2+\gamma_1K^3}{(3\gamma_1K-1)(-1-m^2\pm 6m)}}$$

and $\mathrm{dc}\xi = 1/\mathrm{cd}\xi$. As $m \to 1$, Eq. (18) degenerates to Eq. (17).

Case 3 $p=2-m^2, q=-2, r=-(1-m^2)$. This solution of Eq. (12) is $f(\xi)=\mathrm{dn}\xi$. The periodic wave solution of Eq. (1) reads

$$u = \{\pm \mathrm{dn}[k(x-ct)] \pm \sqrt{1-m^2}\mathrm{nd}[k(x-ct)]\sqrt{\frac{6\gamma_1}{\gamma_2+2\gamma_3}}k\mathrm{e}^{\mathrm{i}(Kx-\Omega t)}, \tag{20}$$

where

$$k=\sqrt{\frac{\Omega-K^2+\gamma_1K^3}{(3\gamma_1K-1)(2-m^2\pm 6\sqrt{1-m^2})}}$$

and $\mathrm{nd}\xi = 1/\mathrm{dn}\xi$. As $m \to 1$, Eq. (20) degenerate to

$$u=\pm \mathrm{sech}[k(x-ct)]\sqrt{\frac{6\gamma_1}{\gamma_2+2\gamma_3}}k\mathrm{e}^{\mathrm{i}(Kx-\Omega t)}, \tag{21}$$

where

$$k=\sqrt{\frac{\Omega-K^2+\gamma_1K^3}{3\gamma_1K-1}}.$$

Case 4 $p=2-m^2, q=2(1-m^2), r=1$. From Eq. (12) we obtains $f(\xi)=\mathrm{sc}\xi=\mathrm{sn}\xi/\mathrm{cn}\xi$. Thus the periodic wave solution of Eq. (1) are

$$u = \{\pm \sqrt{1-m^2}\mathrm{sc}[k(x-ct)] \pm \mathrm{cs}[k(x-ct)]\sqrt{-\frac{6\gamma_1}{\gamma_2+2\gamma_3}}k\mathrm{e}^{\mathrm{i}(Kx-\Omega t)}, \tag{22}$$

where

$$k=\sqrt{\frac{\Omega-K^2+\gamma_1K^3}{(3\gamma_1K-1)(2-m^2\pm 6\sqrt{1-m^2})}}$$

and $\mathrm{cs}\,\xi = 1/\mathrm{sc}\xi$.

According to the extended mapping method, we assume that Eq. (10) has the solution of the form

$$\phi(\xi) = A_0 + A_1 f + B_1 g, \tag{23}$$

where $A_i$ and $B_i$ are constants to be determined, and $f$ and $g$ satisfy the following equations

$$f'^2 = pf^2 + \frac{1}{2}qf^4 + r,$$

$$g'' = g(c_1 + c_2 f^2), g^2 = c_3 + c_4 f^2, \tag{24}$$

where the prime denotes derivative with respect to $\xi$, and $p, q, r$ and $c_i$ are constants to be determined. Thus a new algebraic mapping relation between the solution of Eq. (10) and that of Eq. (24) is established through Eq. (23). The substitution of Eq. (23) into Eq. (10) and the use of Eq. (24) yields (equating the coefficients of like powers of $f^i g^j$ to zero)

$$\begin{cases} f^0: BA_0 + C(A_0^3 + 3c_3 A_0 B_1^2) = 0, \\ g: c_1 AB_1 + BB_1 + C(3A_0^2 B_1 + c_3 B_1^3) = 0, \\ f: pAA_1 + BA_1 + C(3A_0^2 A_1 + 3c_3 A_1 B_1^2) = 0, \\ fg: 6CA_0 A_1 B_1 = 0, \\ f^2: C(3A_0 A_1^2 + 3c_4 A_0 B_1^2) = 0, \\ f^2 g: c_2 AB_1 + 3CA_1^2 B_1 + c_4 CB_1^3 = 0, \\ f^3: qAA_1 + CA_1^3 + 3c_4 CA_1 B_1^2 = 0, \end{cases} \tag{25}$$

from which it is found that

$$A_0 = 0, A_1 = \pm\sqrt{\frac{(c_4 p - c_3 q)A + c_4 B}{c_3 C}}, B_1 = \pm\sqrt{-\frac{pA + B}{3c_3 C}}, \tag{26}$$

$$(3c_4 p - 3c_3 q - c_1 c_4 + c_2 c_3)A + 2c_4 B = 0, (3c_1 - p)A + 2B = 0.$$

Using Eq. (7) and Eq. (9), we obtain the new exact solution of Eq. (6), and it also means the solution of Eq. (1).

$$\begin{aligned} u = [&\pm\sqrt{\frac{(3c_1 c_4 + 2c_3 q - 3c_4 p)(\Omega - K^2 + \gamma_1 K^3)}{(\alpha - \gamma_2 K)c_3(3c_1 - p)}} f(\xi) \\ &\pm\sqrt{-\frac{(c_1 - p)(\Omega - K^2 + \gamma_1 K^3)}{(\alpha - \gamma_2 K)c_3(3c_1 - p)}} g(\xi)] e^{i(Kx - \Omega t)}, \end{aligned} \tag{27}$$

where $f$ and $g$ satisfy Eq. (24) with the constraint among the coefficients

$$4c_4 p - 3c_3 q - 4c_1 c_4 + c_2 c_3 = 0,$$

and

$$\xi = \sqrt{\frac{2(3\gamma_1 K^2 + c - 2K)}{\gamma_1(3c_1 - p)}}(x - ct),$$

where the $K$ and $c$ satisfy Eq. (8), and other parameters are arbitrary constants in the sense that the expressions in the square root should be positive. And note that the choice for the positive and negative signs in Eq. (27) are arbitrary. These settlements are valid throughout the paper. In the following we discuss the specific expressions of $f$ and $g$ according to Eq. (24) as examples.

Case 1 $p = -(1 + m^2), q = 2, r = m^2$

(i) $c_1 = -m^2, c_2 = 2, c_3 = -1, c_4 = 1$. Eq. (24) has the solution $f(\xi) = \mathrm{ns}\,\xi = 1/\mathrm{sn}\,\xi$, $g(\xi) = \mathrm{cs}\,\xi = \mathrm{cn}\xi/\mathrm{sn}\xi$. Thus we get the new periodic wave solution of Eq. (1)

$$u = \pm\sqrt{\frac{\Omega - K^2 + \gamma_1 K^3}{(\alpha - \gamma_2 K)(1 - 2m^2)}}\{\mathrm{ns}[k(x - ct)] + \mathrm{cs}[k(x - ct)]\} e^{i(Kx - \Omega t)}, \tag{28}$$

where $k = \sqrt{\frac{2(3\gamma_1 K^2 + c - 2K)}{\gamma_1(1 - 2m^2)}}$. As $m \to 0$ and $m \to 1$, Eq. (28) degenerate to

$$u = \pm\sqrt{\frac{\Omega - K^2 + \gamma_1 K^3}{\alpha - \gamma_2 K}}\{\csc[k(x - ct)] + \cot[k(x - ct)]\} e^{i(Kx - \Omega t)}, \tag{29}$$

where $k = \sqrt{\frac{2(3\gamma_1 K^2 + c - 2K)}{\gamma_1}}$, and

$$u = \pm\sqrt{-\frac{\Omega - K^2 + \gamma_1 K^3}{\alpha - \gamma_2 K}}\{\coth[k(x - ct)] + \mathrm{csch}[k(x - ct)]\} e^{i(Kx - \Omega t)}, \tag{30}$$

where $k = \sqrt{-\frac{2(3\gamma_1 K^2 + c - 2K)}{\gamma_1}}$, respectively.

(ii) $c_1=-1, c_2=2, c_3=-m^2, c_4=1$. The solution of Eq. (24) reads $f(\xi)=\mathrm{ns}\xi, g(\xi)=\mathrm{ds}\xi=\mathrm{dn}\xi/\mathrm{sn}\xi$. Thus another new periodic wave solution of Eq. (1)

$$u=\pm\sqrt{\frac{\Omega-K^2+\gamma_1K^3}{(\alpha-\gamma_2K)(-2+m^2)}}\{\mathrm{ns}[k(x-ct)]+\mathrm{ds}[k(x-ct)]\}\mathrm{e}^{\mathrm{i}(Kx-\Omega t)}, \tag{31}$$

where $k=\sqrt{\dfrac{2(3\gamma_1K^2+c-2K)}{\gamma_1(-2+m^2)}}$.

(iii) $c_1=-1, c_2=2, c_3=-\dfrac{m^2}{1-m^2}, c_4=\dfrac{1}{1-m^2}$. The solution of Eq. (24) is $f(\xi)=\mathrm{dc}\xi=\mathrm{dn}\xi/\mathrm{cn}\xi, g(\xi)=\mathrm{nc}\xi=1/\mathrm{cn}\,\xi$. Therefore, we have

$$u=\pm\sqrt{\frac{\Omega-K^2+\gamma_1K^3}{(\alpha-\gamma_2K)(-2+m^2)}}\{\mathrm{dc}[k(x-ct)]+\sqrt{1-m^2}\,nc[k(x-ct)]\}\mathrm{e}^{\mathrm{i}(Kx-\Omega t)}, \tag{32}$$

where $k=\sqrt{\dfrac{2(3\gamma_1K^2+c-2K)}{\gamma_1(-2+m^2)}}$.

(iv) $c_1=-m^2, c_2=2, c_3=-\dfrac{1}{1-m^2}, c_4=\dfrac{1}{1-m^2}$. The solution of Eq. (24) is $f(\xi)=\mathrm{dc}\xi, g(\xi)=\mathrm{sc}\xi=\mathrm{sn}\xi/\mathrm{cn}\xi$. Therefore, we get

$$u=\pm\sqrt{\frac{\Omega-K^2+\gamma_1K^3}{(\alpha-\gamma_2K)(1-2m^2)}}\{\mathrm{dc}[k(x-ct)]+\sqrt{1-m^2}\,\mathrm{sc}[k(x-ct)]\}\mathrm{e}^{\mathrm{i}(Kx-\Omega t)}, \tag{33}$$

where $k=\sqrt{\dfrac{2(3\gamma_1K^2+c-2K)}{\gamma_1(1-2m^2)}}$.

Case 2 $p=2m^2-1, q=-2m^2, r=1-m^2, c_1=m^2, c_2=-2m^2, c_3=1-m^2, c_4=m^2$.

In this case, we have $f(\xi)=\mathrm{cn}\xi, g(\xi)=\mathrm{dn}\xi$. Thus the new periodic wave solution of Eq. (1) is

$$u=\pm\sqrt{-\frac{\Omega-K^2+\gamma_1K^3}{(\alpha-\gamma_2K)(1+m^2)}}\{m\mathrm{cn}[k(x-ct)]+\mathrm{dn}[k(x-ct)]\}\mathrm{e}^{\mathrm{i}(Kx-\Omega t)}, \tag{34}$$

where $k=\sqrt{\dfrac{2(3\gamma_1K^2+c-2K)}{\gamma_1(1+m^2)}}$. As $m\to 1$, Eq. (34) degenerates to

$$u=\pm\sqrt{\frac{\Omega-K^2+\gamma_1K^3}{-2(\alpha-\gamma_2K)}}\mathrm{sech}[k(x-ct)]\mathrm{e}^{\mathrm{i}(Kx-\Omega t)}, \tag{35}$$

where $k=\sqrt{\dfrac{3\gamma_1K^2+c-2K}{\gamma_1}}$.

Case 3 $p=2m^2-1, q=2(1-m^2), r=-m^2, c_1=m^2, c_2=1-2m^2, c_3=-1, c_4=1$.

In this case, Eq. (24) has the solution $f(\xi)=\mathrm{nc}\xi, g(\xi)=\mathrm{sc}\xi$. Therefore, we have

$$u=\pm\sqrt{\frac{(1-m^2)(\Omega-K^2+\gamma_1K^3)}{(\alpha-\gamma_2K)(1+m^2)}}\{\mathrm{nc}[k(x-ct)]+\mathrm{sc}[k(x-ct)]\}\mathrm{e}^{\mathrm{i}(Kx-\Omega t)}, \tag{36}$$

where $k=\sqrt{\dfrac{2(3\gamma_1K^2+c-2K)}{\gamma_1(1+m^2)}}$. As $m\to 0$, Eq. (36) degenerates to

$$u=\pm\sqrt{\frac{\Omega-K^2+\gamma_1K^3}{\alpha-\gamma_2K}}\{\sec[k(x-ct)]+\tan[k(x-ct)]\}\mathrm{e}^{\mathrm{i}(Kx-\Omega t)}, \tag{37}$$

where $k=\sqrt{\dfrac{2(3\gamma_1K^2+c-2K)}{\gamma_1}}$.

## 3. Conclusion

In conclusion, the modified and extended mapping method has been proposed to obtain the new exact solution of the high order nonlinear Schrödinger equation. The new exact solution include the periodic wave solution in terms of the Jacobi elliptic functions, the triangular periodic wave solution and the solitary wave solutions, whether this equation possesses other type of solution by changing the conditions is worthy of studying further.

## References:


[1] Matveev V B. Salle M A. Darboux Transformations and Solitons[M]. Berlin: Springer, 1991.

[2] Fan En-gui. A series of new exact solutions for a complex coupled KdV system[J]. Chaos. Solitons and Fractals, 2004, 19(3): 515～525.

[3] Malfliet W. The tanh method: a tool for solving certain classes of nonlinear evolution and wave equations [J]. Journal of Computational and Applied Mathematics, 2004, 164(1): 529～541.

[4] Soliman A A. The modified extended tanh-function method for solving Burgers-type equations[J]. Physica A, 2006, 361(2): 394～404.

[5] Lu Da-Zhao. Abundant Jacobi elliptic function solutions of nonlinear evolution equations[J]. Acta Physica Sinica, 2005, 54(10): 4501～4505.

[6] Liu Shi-kuo. Jacobi elliptic function expansion solution to the variable coefficient nonlinear equations[J]. Acta Physica Sinica, 2002, 51(9): 1925～1926.

[7] Yan Zhen-ya. The Weierstrass elliptic function expansion method and its applications in nonlinear wave equations[J]. Chaos. Solitons and Fractals, 2006, 29(4): 948～964.

[8] Abramowitz M, Stegun I A. Handbook of Mathematical Functions[M]. New York: Dove, 1972.

[9] Peng Yan-ze. Exact periodic wave solutions to a new Hamitonian amplitude equation[J]. Journal of the Physical Society of Japan, 2003, 72(6): 1356～1359.

[10] Peng Yan-ze. New exact solutions to a new Hamiltonian amplitude equation I [J]. Journal of the Physical Society of Japan, 2003, 72(8): 1889～1890.

[11] Peng Yan-ze. New exact solutions to a new Hamiltonian amplitude equation II [J]. Journal of the Physical Society of Japan, 2004, 73(5): 1156～1158.

[12] Gong Lun-xun. Same new exact solutions of the Jacobi elliptic functions of NLS equation[J]. Acta Physica Snica, 2006, 55(9): 4414～4419.


# 高阶非线性薛定谔方程的精确解


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**摘要**：通过修正的映射方法和推广的映射方法，我们得到了高阶非线性薛定谔方程新的精确解，它们是两个不同的雅可比椭圆函数的线性组合. 并研究了在极限情况下高阶非线性薛定谔方程的解.



**关键词**：精确解；高阶非线性薛定谔方程；修正的映射方法；推广的映射方法

# 高阶非线性薛定谔方程的精确解



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刊名： 应用数学 ISTIC PKU
英文刊名： MATHEMATICA APPLICATA
年，卷(期)： 2007,20(3)
被引用次数： 1次

## 参考文献(12条)

1. Malfliet W The tanh method:a tool for solving certain classes of nonlinear evolution and wave equations 2004(01)

2. Fan En-gui A series of new exact solutions for a complex coupled KdV system[外文期刊] 2004(03)

3. Matveev V B;Salle M A Darboux Transformations and Solitons 1991

4. Gong Lun-xun Same new exact solutions of the Jacobi elliptic functions of NLS equation[期刊论文]-Acta Physica Sinica 2006(09)

5. Peng Yan-ze New exact solutions to a new Hamiltonian amplitude equation II[外文期刊] 2004(05)

6. Peng Yan-ze New exact solutions to a new Hamiltonian amplitude equation I[外文期刊] 2003(08)

7. Peng Yan-ze Exact periodic wave solutions to a new Hamitonian amplitude equation[外文期刊] 2003(06)

8. Abramowitz M;Stegun I A Handbook of Mathematical Functions 1972

9. Yan Zhen-ya The Weierstrass elliptic function expansion method and its applications in nonlinear wave equations[外文期刊] 2006(04)

10. Liu Shi-kuo Jacobi elliptic function expansion solution to the variable coefficient nonlinear equations[期刊论文]-Acta Physica Sinica 2002(09)

11. Lu Da-Zhao Abundant Jacobi elliptic function solutions of nonlinear evolution equations[期刊论文]-Acta Physica Sinica 2005(10)

12. Soliman A A The modified extended tanh-function method for solving Burgers-type equations[外文期刊] 2006(02)

## 引证文献(1条)

1. 高秀云.段文山 非线性薛定谔方程的新精确解[期刊论文]-西北师范大学学报(自然科学版) 2008(1)